



Ion-Acoustic Double Layers in Dusty Plasma Consisting of Two-Temperature Nonisothermal Electrons and Positrons

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Abstract

Ion-acoustic double layers in a multicomponent dusty plasma consisting of two-temperature nonisothermal electrons, negatively charged dust grains, warm positive ions and positrons has been theoretically studied using pseudo-potential technique. The study shows that the presence of both nonisothermal two-temperature electrons and positrons have significant effects on the excitation and structure of ion-acoustic double layers in the dusty plasma under consideration. Conditions for the existence of double layers are obtained and the critical density of positrons being identified which has significant role on the formation and nature of ion-acoustic double layers in the dusty plasma. The profiles of double layers in the dusty plasma are drawn for different values of the plasma parameters and their effects have been discussed. The importance of the results in astrophysical plasma has been discussed.

Key Words: Double layers, dusty plasma, positrons, nonisothermal two-temperature electrons.

Introduction

The double layers in plasma play a very important and fundamental role in astrophysical phenomena [1,2]. In particular, double layers are responsible for accelerating particles in auroral region of the ionosphere [3]. They may also have important roles in many cosmic sites including the solar atmosphere [4,5], the magnetosphere of the Earth and of Jupiter [6]. In recent years, the study of propagation of waves including double layers in electron-ion-positron plasma has received considerable interest because of its important role in the understanding of the nonlinear wave features in the laboratory and space plasmas [7-10]. It is believed that electron-positron plasma has its origin in the early stage of universe [11-13] and this has important contribution on the physical processes in galactic nuclei [14], pulsar magnetosphere [15], polar caps of neutron stars [16]. Studies on the ion on nonlinear propagation of waves in a plasma consisting of electrons and positrons have been carried out by various authors [17-21] and it has been found that positrons in the plasma have important roles on the phenomena in astrophysical plasma. Moreover, the presence of charged dust particles in plasma has been found to be important because it helps to understand various linear and nonlinear phenomena in cometary tails, planetary rings, asteroids, magnetosphere, lower ionosphere, interstellar and circumstellar clouds, laboratory devices etc. Propagation of dust-acoustic wave (DAW) has been studied theoretically by



Rao et al [22], with the dust grains providing the inertia and the pressure of inertialess electrons and ions providing the restoring force. In last few years, Several authors have theoretically and experimentally studied solitary waves and double layers considering both dust acoustic and ion acoustic waves in magnetized and unmagnetized plasma [23-27].

In the study of ion-acoustic waves and double layers in isothermal plasma one usually considers Boltzmann distributed electrons. But, when the amplitude of wave is large, the electrons may be trapped in the potential trough [28,29]. In fact such trapping can occur even for small amplitude ion acoustic waves. These trapped electrons interact strongly with the wave during the evolution of the wave and it cannot be treated on the same footing as the free electrons. The distributions of trapped electrons are found during the condensation of dust grains in dusty plasmas. It is also worthy to mention that the electrons in plasma may exist at two different temperatures [32]. The presence of these two-temperature electrons in plasma gives rise to many interesting characteristics in nonlinear propagation of waves including the excitation of ion-acoustic solitary waves and double layers in plasma [33-38]. Considering the effects of trapped electrons, the nonlinear wave structures in dusty plasma have been studied by several authors [39,40].

However, double layers in dusty plasma are less studied although it is very much important in the context of astrophysical plasma. So, in the present paper we are interested to study theoretically the ion acoustic double layers in a dusty plasma consisting of warm ions, positrons, cold negatively charged grains and nonisothermal two-temperature electrons. In Sec 2, the basic equations with some assumptions are given for the dusty plasma. In Section 3, nonlinear equation is derived and its double layer solution of ion-acoustic in plasma with positive ions, negative dust particles and two temperature nonisothermal electrons are obtained. In Sec.4, we analyze the results graphically for a dusty plasma having two temperature nonisothermal electrons.

2. Basic Equations

We assume the plasma is unmagnetized, collisionless and consists of warm ions, positrons, non-isothermal two-temperature electrons and cold dust grains. The dust grains have uniform mass and behave like point charges. Moreover, we assume the charging of the dust particles is mainly caused by the attachment of the ions and electrons to the dust grains via collisions. The effects of photo-ionization radiation etc. for charging of the dust particles are neglected. Electrons because of their lighter mass and faster motion are initially attached to the dust particles at a faster rate than the ions, as a result the dust particles get negatively charged.

The non-dimensional equations which do govern the dynamics of the ion and the dust particles are:



For the warm ions:

$$\frac{\partial n_i}{\partial t} + \frac{\partial(n_i u_i)}{\partial x} = 0 \quad (1)$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} + \frac{3\sigma_i}{(1 + \chi_d Z_d)^2} n_i \frac{\partial n_i}{\partial x} = -\frac{\partial \phi}{\partial x} \quad (2)$$

For the cold dust grains:

$$\frac{\partial n_d}{\partial t} + \frac{\partial(n_d u_d)}{\partial x} = 0 \quad (3)$$

$$\left(\frac{\partial}{\partial t} + u_d \frac{\partial}{\partial x}\right) u_d = \left(\frac{Z_d}{\mu_d}\right) \frac{\partial \phi}{\partial x} \quad (4)$$

The Poisson equation is

$$\frac{\partial^2 \phi}{\partial x^2} = n_{el} + n_{eh} - n_i - n_p + Z_d n_d \quad (5)$$

where, $\sigma_i = T_i / T_{eff}$, $\chi_d = n_{d0} / n_0$ where T_i , T_{ef} , n_0 , n_{d0} denote the ion temperature, effective electron temperature, the equilibrium values of the background electron-density, the number-density of the dust particles respectively at equilibrium state. Z_d is charge number in the dust particles. $\mu_d = m_i / m_d$, m_i and m_d being the mass of the ion and dust particles. u_i and u_d are the velocities of ions and dust particles respectively. n_i , n_d , n_{eh} , n_{el} are the densities of ion, dust particles, hot electrons and cold electrons respectively. Φ is the electrostatic potential. In all these equations, the velocities are normalized by $(K_B T_{ef} / m_i)^{1/2}$, K_B being the Boltzman constant.

The electron densities n_{el} and n_{eh} at low and high temperatures under non-isothermal conditions can be assumed as [41]

$$n_{el} = \mu \left[\exp\left(\frac{T_{ef}}{T_{el}} \phi\right) \operatorname{erfc} \exp\left(\frac{T_{ef}}{T_{el}} \phi\right)^{1/2} + \frac{1}{\sqrt{\beta_l}} \exp\left(\beta_l \frac{T_{ef}}{T_{el}} \phi\right) \operatorname{erfc} \exp\left(\beta_l \frac{T_{ef}}{T_{el}} \phi\right)^{1/2} \right] \quad (6a)$$

$$n_{eh} = \nu [\exp(\frac{T_{ef}}{T_{eh}} \phi) \operatorname{erfc} \exp(\frac{T_{ef}}{T_{eh}} \phi)^{1/2} + \frac{1}{\sqrt{\beta_h}} \exp(\beta_h \frac{T_{ef}}{T_{eh}} \phi) \operatorname{erfc} \exp(\beta_h \frac{T_{ef}}{T_{eh}} \phi)^{1/2}] \quad (6b)$$

where, the effective temperature of the plasma $T_{eff} = T_{el} T_{eh} / (\mu T_{eh} + \nu T_{el})$, $\beta_l = T_{el} / T_{elt}$, $\beta_h = T_{eh} / T_{eht}$, T_{elt} and T_{eht} are the temperatures of trapped electrons in the low and high temperature groups of electrons. μ and ν are the equilibrium number density of low and high temperature electrons.

Expanding in ascending power of ϕ we obtain the electron densities at low and high temperatures as

$$n_{el} = \mu [\exp(\frac{T_{ef}}{T_{el}} \phi) \operatorname{erfc} \exp(\frac{T_{ef}}{T_{el}} \phi)^{1/2} + \beta_l^{-1/2} \exp(\beta_l \frac{T_{ef}}{T_{el}} \phi) \operatorname{erfc} \exp(\beta_l \frac{T_{ef}}{T_{el}} \phi)^{1/2}] \quad (7a)$$

$$n_{eh} = \nu [1 + \frac{\beta \phi}{Y} - \frac{4}{3} b_h (\frac{\beta \phi}{Y})^{3/2} + \frac{1}{2} b_h (\frac{\beta \phi}{Y})^2 - \frac{8}{15} b_h^{(1)} (\frac{\beta \phi}{Y})^{5/2} + \frac{1}{6} (\frac{\beta \phi}{Y})^3 + \dots] \quad (7b) \quad \text{where,}$$

$$\beta = T_{el} / T_{eh}$$

$$b_l = \frac{1 - \beta_l}{\sqrt{\pi}}, b_h = \frac{1 - \beta_h}{\sqrt{\pi}}, b_l^{(1)} = \frac{1 - \beta_l^2}{\sqrt{\pi}}, b_h^{(1)} = \frac{1 - \beta_h^2}{\sqrt{\pi}}, Y = \mu + \nu \beta$$

The density of the positrons is given by

$$n_p = \chi_p \exp(-\sigma_p \phi) \quad (8)$$

Where,

$$\chi_p = n_{p0} / n_{e0} = 1 - n_{i0}, \sigma_p = T_e / T_p.$$

3. Nonlinear Equation and Double layer solution.

For the study of double layers for the ion-acoustic wave in dusty plasma, we have to derive a nonlinear equation for the dynamics of plasma particles. To do this, the equations (1) - (5) are transformed, on introducing the Galilean transformation involving a new variable, ξ given by $\xi = x - M t$ we get after using the boundary conditions

$$\phi \rightarrow 0, n_p, n_i \rightarrow (1 + \chi_d Z_d), n_d \rightarrow \chi_d,$$

$$u_i \rightarrow 0, u_d \rightarrow 0 \quad (9)$$

$$\text{as } \xi \rightarrow \infty$$

the ion density and dust density are obtained as

$$n_i = \frac{(1 + \chi_d Z_d)}{(1 - \frac{2\phi}{M^2 - 3\sigma_i})^{1/2}} \quad (10)$$

$$n_d = \frac{\chi_d Z_d}{(1 + \frac{Z_d \phi}{\mu_d M^2})^{1/2}} \quad (11)$$

Using (7a), (7b), (7c), (9) and (10) in (5) and expanding in power series of ϕ we obtain

$$\frac{\partial^2 \phi}{\partial \xi^2} = A_1 \phi - A_2 \phi^{3/2} + A_3 \phi^2 + A_4 \phi^{5/2} + A_5 \phi^3 + \dots \quad (12a)$$

$$= -\frac{d\psi}{d\phi} \quad (12b)$$

where

$$A_1 = -\frac{1}{8} [P_1(1 + \alpha Z_d) + P_2 \alpha Z_d] + \chi_p \sigma_p, \quad (13a)$$

$$A_2 = -\frac{4}{15} \left(\frac{\mu b_l + \nu b_h \beta^{3/2}}{Y^{3/2}} \right) \quad (13b)$$

$$A_3 = \left[\frac{1}{16} \{P_1^2(1 + \alpha Z_d) - P_2^2 \alpha Z_d\} - \frac{1}{12} \left(\frac{\mu b_l + \nu b_h \beta^2}{Y^2} \right) \right] + \frac{1}{2} \chi_p \sigma_p^2 \quad (13c)$$

$$A_4 = -\frac{8}{105} \left(\frac{\mu b_l^{(1)} + \nu b_h^{(1)} \beta^{5/2}}{Y^{5/2}} \right) \quad (13d)$$

$$A_5 = \left[\frac{5}{128} \{P_1^3(1 + \alpha Z_d) + P_2^3 \alpha Z_d\} - \frac{1}{48} \left(\frac{\mu + \nu \beta^3}{Y^3} \right) \right] + \frac{1}{6} \chi_p \sigma_p^3 \quad (13e)$$

$$P_1 = \frac{2}{M^2 - 3\sigma_i}, \quad (13f)$$

$$P_2 = \frac{2Z_d}{\mu_d M^2} \quad (13g)$$

and $\psi(\varphi)$ is the Sagdeev potential [42].

To obtain the double layer solution of (12b) in lowest order, we retain the terms up to φ^2 in (12a). Therefore, integrating Eq.(12b), we get

$$\frac{1}{2} \left(\frac{\partial \varphi}{\partial \xi} \right)^2 + \psi_1(\varphi) = 0 \quad (14)$$

where

$$\psi_1(\varphi) = -\frac{A_1}{2} \varphi^2 + \frac{2A_2}{5} \varphi^{5/2} - \frac{A_3}{3} \varphi^3 \quad (15)$$

Now, substituting $\varphi = \Phi^2$ in Eq.(14) we obtain

$$2 \left(\frac{\partial \Phi}{\partial \xi} \right)^2 = \frac{A_1}{2} \Phi^2 - \frac{2A_2}{5} \Phi^3 + \frac{A_3}{3} \Phi^4 \quad (16)$$

$$= -\psi_1(\Phi)$$

To get the double layer solution from the Eq.(16) the modified Sagdeev potential $\psi(\Phi)$ must satisfy the following boundary conditions:

i) $\psi_1(\Phi) \rightarrow 0$ as $\Phi \rightarrow 0$ and $\Phi \rightarrow \Phi_m$,

ii) $d\psi_1/d\Phi \rightarrow 0$ as $\Phi \rightarrow \Phi_m$

and $\Phi \rightarrow 0$, (17)

and

iii) $d^2\psi_1/d\Phi^2 \rightarrow 0$ as $\Phi \rightarrow 0$

and $\Phi \rightarrow \Phi_m$,



where, Φ_m is the maximum value of Φ .

Using above boundary conditions (17), we find that

$$A_1 = \frac{4A_2}{5}\Phi_m - \frac{2A_3}{3}\Phi_m^2$$

and

$$A_1 = \frac{4A_2}{5}\Phi_m - \frac{2A_3}{3}\Phi_m^2 \quad (18)$$

From the above two relations A_1 and A_2 are evaluated as

$$A_1 = \frac{2A_3}{3}\Phi_m^2,$$

$$A_2 = \frac{5A_3}{3}\Phi_m$$

and

$$25A_1A_3 = 6A_2^2 \quad (19)$$

Now, putting the values of A_1 and A_2 in (16) we get

$$\frac{d\Phi}{d\xi} = F\Phi(\Phi_m - \Phi) \quad (20)$$

where, $F = \pm\left(\frac{A_3}{6}\right)^{1/2}$

Now, we use a hyperbolic transformation $z = \tanh(\xi)$ and $W(z) = \varphi(\xi)$ in Eq.(20) and obtain

$$(1-z^2)\frac{dW}{dz} - F\Phi_m W + FW^2 = 0 \quad (21)$$

Eq.(21) is a Fuchsian-like nonlinear ordinary differential equation which gives a Forbenius series solution as:

$$W(z) = \sum_{r=0}^N a_r z^{\rho+r} \quad (22)$$

where ρ determines the number and nature of solution. Substituting (22) in (21) and balancing the leading order of the nonlinear terms with the order of the differential equation we obtain $N=1$ i.e. the series $W(z)$ would have two terms. So, the series (22) become,

$$W(z) = a_0 + a_1 z \quad (23)$$

Using (23) we obtain from (21)

$$\begin{aligned} -a_1 + Fa_1^2 &= -Fa_1\Phi_m + 2Fa_0a_1 \\ &= -1 - F\Phi_m a_0 + Fa_0^2 \end{aligned} \quad (24)$$

Eq. (21) leads to determine the unknowns parameters a_0 , a_1 and F as:

$$a_0 = \pm \frac{1}{\Phi_m}, a_1 = \frac{\Phi_m}{2} \text{ and } F = \frac{2}{\Phi_m} \quad (25)$$

The double-layer solution of (17) is obtained as ,

$$\varphi_{D1}(\xi) = \frac{\phi_{01}^2}{4} \left[1 - \tanh\left(\frac{\phi_{01}\xi}{\sqrt{-16/A_3}}\right) \right]^2 \quad (26)$$

where, $\phi_{01} = \frac{A_2}{2A_3}$.

From (26) it is seen that the double layers exist if $A_3 > 0$. Since A_2 and A_3 of (13a) and (13b) are dependent on the concentration of two-temperature electrons, nonisothermal parameter, dust density etc. So, φ_0 , the amplitude of the double layers, also depend on these parameters.

The thickness of double layers is derived as

$$d_1 = \frac{\left(\frac{-16}{A_3}\right)^{1/2}}{\phi_{01}} \quad (27)$$

4. Results and Discussions

From (26), it is seen that amplitudes of the DLs depend on the dust density, positron density, nonisothermality of two-temperature electrons, density of the electrons at two different temperatures, ion temperature etc. The positive values of φ_{01} , for some values of the plasma parameters, give compressive DLs; on the other hand, the negative values of φ_{01} give the rarefactive double layers for some other values of the plasma parameters. To understand the behaviour of the DLs, the profiles DLs are drawn in Figs.1-5 using data of a model dusty plasma consisting of positrons and nonisothermal two-temperature electrons. In Fig.1, the profiles of DL's are shown with variation of dust density in a plasma having $\sigma=0.001$, $M=1.8$,

$\mu=0.15$, $Z_d=10^3$, $bl=0.3$, $bh=0.5$, $\chi_p=0.1$, $\sigma_p=0.1$, $\mu_d=10^{10}$. It is seen that the double layers are compressive in nature and the amplitude increases with the increase of dust density.

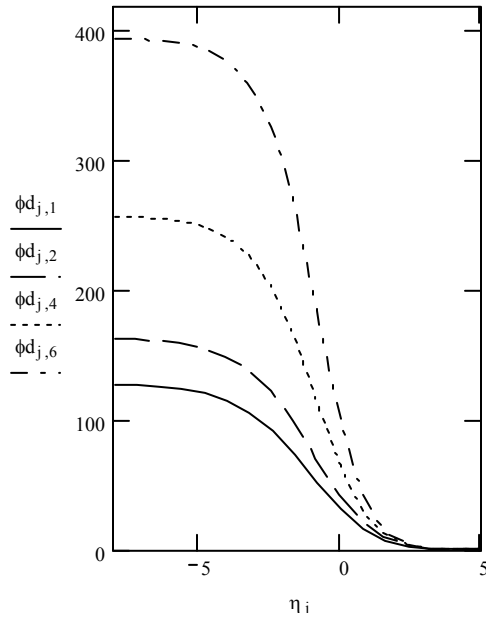


Fig.1: The profiles of DLs for different values of dusty density plasma. The graphs from bottom to top showed by solid, dashed and dotted lines correspond to $nd_0 = 0.0001$, 0.0002 , 0.0004 and 0.0006 respectively.

The effects of the temperature ratio of low and high temperature electrons (β) on double layers are shown in Fig.2 in dusty plasma having $\sigma_i=0.01$, $M=2.5$, $\mu=0.15$, $nd_0=0.001$, $Z_d=10^3$, $bl=0.15$, $bh=0.5$, $\chi_p=0.1$, $\sigma_p=0.1$, $\mu_d=10^6$. Fig.2 shows that double layers are also compressive in nature and amplitudes are decreased with the increase of β .

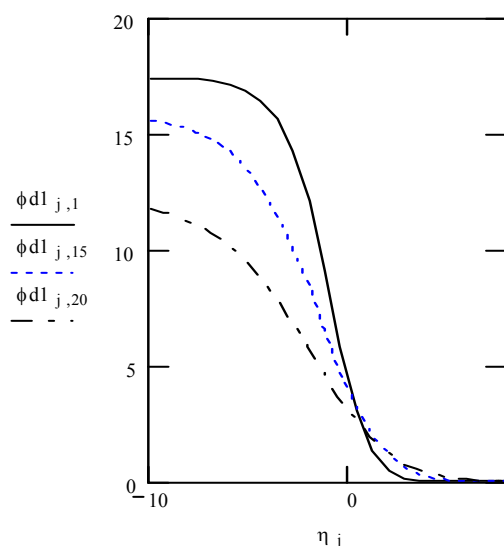


Fig.2: The profiles of DLs for different values of β in dusty plasma. The graphs from top to bottom showed by solid, dotted and dashed lines correspond to $\beta = 0.02, 0.3$ and 0.4 respectively.

The effects of electron densities at low and high temperatures are shown in Fig.3 for the dusty plasma having $\beta = 0.2$, $\sigma_i = 0.001$, $M = 2.5$, $\chi_p = 0.1$, $\sigma_p = 0.1$, $nd_0 = 0.001$, $Z_d = 1000$, $bl = 0.15$, $bh = 0.4$, $nd_0 = 0.001$, $Z_d = 10^3$, $\mu_d = 10^6$. It has been found that amplitude of the compressive DL's is decreased with increase of the electron density.

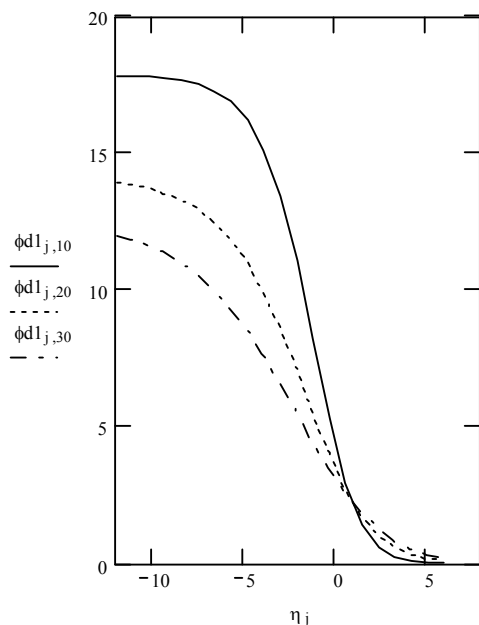


Fig.3. Profiles of DLs for different values of electron density(μ) in dusty plasma. The graphs from top to bottom showed by solid, dotted and dash-dotted lines correspond to $\mu = 0.15, 0.3$ and 0.45 respectively.

The profiles of double layers for different values of positron density (χ_p) in dusty plasma having $\sigma_i = 0.01, M = 2.5, \mu = 0.15, n_{d0} = 0.001, Z_d = 1000, b_l = 0.15, b_h = 0.5, \chi_d = 0.2, \sigma_p = 0.1, \mu_d = 10^6$ are shown in Fig.4 from which it is seen that amplitude of double layers increases with the increase of positron density.

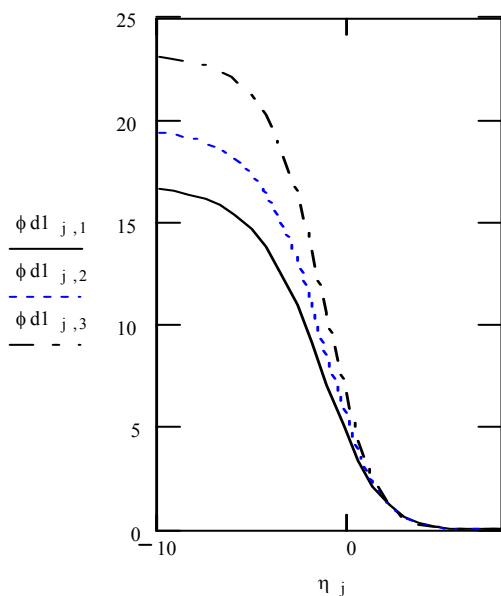


Fig.4: Profiles of DLs for different values of positron density (χ_p) in dusty plasma. The graphs from bottom to top showed by solid, dotted and dashed lines correspond to $\chi_p = 0.2, 0.4$ and 0.6 respectively.

The effect of positron temperature (σ_p) on the double layers in dusty plasma having $\sigma = 0.01, M = 2.5, \mu = 0.15, n_{d0} = 0.001, Z_d = 1000, b_l = 0.15, b_h = 0.5, \chi_d = 0.2, \chi_p = 0.2, \mu_d = 10^6$ are shown in Fig.5. It is seen that amplitude of double layers increase with the increase of positron temperature in the dusty plasma.

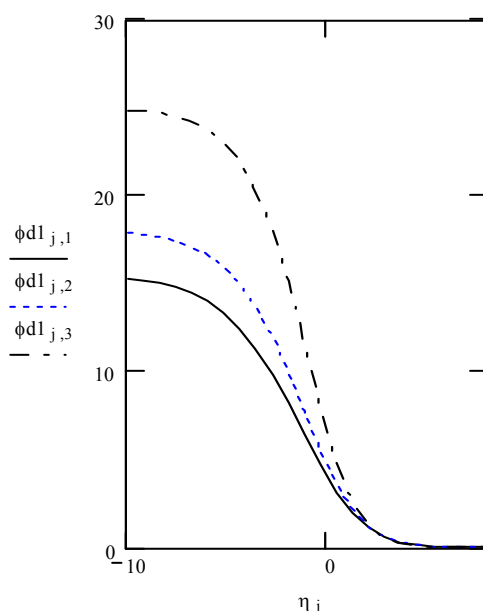


Fig.5. Profiles of DLs for different values of positron temperature (σ_p) in dusty plasma., The graphs from bottom to top showed by solid, dotted and dash-dotted lines correspond to $\sigma_p = 0.05, 0.1$ and 0.15 respectively.

Summary and concluding Remarks

In this paper, the DLs in a dusty plasma consisting of warm ions, positrons, nonisothermal electrons and negatively charged dust particles have been theoretically investigated using the tanh-method. It is seen that compressive double layers will be excited in the dusty plasma depending upon the concentration of dust particles, positrons, nonisothermal parameter of electrons, temperatures of the ions and positrons etc. It is known that double layers are formed due to net potential difference of electrostatic fields and any particle traveling through the region of double layers is directly associated by this potential differences. So, our present paper on the double-layers gives the idea about the possible source of acceleration of charged particles in dusty plasma of the astrophysical objects.

To study the double layers in the dusty plasma we have assumed that charges of the dusty particles are constant. But, the dust charges are variable because electrons and ions are continuously being attached with the dust particle through collisions [41]. Moreover, in astrophysical objects, magnetic field is present and it has important roles on the nonlinear phenomena in the plasma. Gravitational effect in dusty plasma may also have significant effect on the excitation of ion acoustic double layers like ion acoustic solitary waves [42,43]. Our works on the double layers considering the variable dust charges and magnetic field are in progress which would be communicated in near future.



References

- [1]H. Alfven, Cosmical electrodynamics, ClarendonPress, Oxford(1950)
- [2]H.Alven, Cosmic Plasma, D.Reidal Publishing Company,Dardrecht, Holland, Chapter-II, pp.29-41 (1981)
- [3]H.Alfven, Physics Today, **39(9)**, 22(1986)
- [4]P. Catlquist, Astrophys. Space Sc.,**87**, 21(1982)
- [5]S. Troven, L.Lindberg and R. Carpenter, Plasma Phys. **27**,143(1985)
- [6]S.D. Shawan, J.Geophys.Res.**81**,3373(1976)
- [7]. N L Satalishvili, J L Javakeshvili and H Kaya *Astrophys.Space Sci.* 250 109 (1997)
- [8]. P.K.Shukla and L.Stenflo, *Astrophys. Space Sci.* , 209 323 (1993).
- [9]. S.Mahmood,A.Mustaq and H.Saleem, *New J. Phys.* 5 28(2003)
- [10]. H.Saleem, *Phys. Plasmas*, 13(3) 034503 (2006)
- [11]. W.Misner, K.S.Thorne and J.A.Wheeler , *Gravitation,Freeman*, San Francisco(1980).
- [12].M.J.Rees, G.W.Gibbons,S.W. Hawkins and S. Siklas, (editors)*The very earlier universe*, Cambridge Universal Press (1983).
- [13]. Y.B.Zeldovich and I.Novikov, *Relativistic Astrophysics*, University of Chicago,Chicago, 1983.
- [14].H.R.Millar and P.J. Wilte, *Active Galactic Nuclei*, Springer-Verlag, Berlin, 202 (1987).
- [15]. P.A.Sturrock and K B Baker, *Astrophys. J.* 234 612 (1979).
- [16]. F.C.Michel, *Theory of Neutron Star Magneto-sphere*, Chicago Univ. Press, Chicago, (1991).
- [17]. R. Pillay and R. Bharuthram, *Astrophys. Space Sci.* 188 592 (1992).
- [18]. S.Burman and A.Roychowdury, *Fizika* A8 83 (1999).
- [19]. H.Alinejad, S.Sovanian and J.Mahmoodi, *Phys.Plasmas* 13 012304 (2006).
- [20]. W.Massod,N.Jehrab,A.M.Mirza and P.H.Sakanaka,*Phys.Lett. A*372 4279 (2008).
- [21]. S. N. Paul, C.Das, I.Paul, B. Bandyopadhyay, S.Chattopadhyay and S .S. De ,*Indian J.Phys.*,Vol. 86 (6), pp. 545-553 (2012).
- [22].N.N. Rao, P.K.Shukla and M.Y. Yu, *Planet Space Sci.* **38**,543 (1990)
- [23].J.X. Ma and L. Liu, *Phys. Plasmas*, **4**, 253(1997).
- [24].F. Verheest, *Phys. Plasmas*, **.5**, 63 (1998).
- [25].S.N. Paul, K.K.Mondal and A. Roychowdhury,*Phys. Lett. A.*, **Vol. 257**, 165 (1999).
- [26].G .C. Das, J Sarma and C Uberoi, *Phys. Plasmas* **4**, 2095 (1997)
- [27].F.Verheest, *Waves in dusty plasma*, Kluwer Academic Publications, Netherlands, (2000)
- [28].H. Schamel, *Plasma Phys.***14**, 905(1972)
- [29].H. Schamel, *J.Plasma Phys*, **9**,377 (1973)



- [30].D.Jones,A. Lee, S.M. Gleeman and H.J.Doucet, *Phys.Rev.Lett.* **35**, 1349 (1975)
- [31].Y Nejah, *Aust. J. Phys.* **51**, 95 (1998)
- [32].G. C Das, J Sarma and M Talukdar, *Phys. Plasmas* **5**, 63 (1998)
- [33].G.C. Das, S.N Paul, and B. Karmakar, *Phys. Fluids*,**29**,2192(1986)
- [34].K.P .Das, S.R. Majumdar and S.N. Paul, *Phys. Rev.*, . **E51**, 4776 (1995).
- [35].S.R.Majumdar,A.Roychowdhury, S.K.Bhattacharyya and **S.N.Paul** , *J. Plasma Phys.* ,**47**,pp.451 (2004)
- [36].S.R.Majumdar, A.Roychowdhury and **S.N.Paul**, *Physica Scripta*, **69**,355 (2004)
- [37]. G. C. Das and J. Sarma *Phys. Plasma*, **5**,3918 (1998)
- [38] .O. Rahman, M. D. I. Bhuyan, M. Haidar and J Islam *International J. Astron. and Astrophys.*,**4**,119 (2014)
- [39]. S.R.Majumdar, S.N.Paul and K.P.Das, *J.Plasma Phys.* **64**,297 (2000)
- [40]. R.Z.Sagdeev, *Rev. Plasma Phys.* edited by M A Leontovich, New York (Consutant Bureau), **4**, 23 (1966)
- [41]. R .K .Varma, P. K. Shukla and V. Krishan, *Phys. Rev.* **E47**, 3612 (1993)
- [42]. S N Paul, K Roychowdhury, S Burman, A Roychowdhury and B Paul *Czech. J. Phys.* **56**,1453 (2006).
- [43] S N Paul, G Pakira, B Paul and B Ghosh *Acta Physica polonica* **A 122**, 116 (2012)