



## Analytical Study on Emission of Strong Terahertz Pulses From Laser Wakefields in Plasma

DIVYA SINGH<sup>1,2\*</sup>, HITENDRA K. MALIK<sup>1</sup>

<sup>1</sup>Department of Physics, PWAPA Laboratory, Indian Institute of Technology Delhi, New Delhi - 110016, India.

<sup>2</sup> Department of Physics & Electronics, Rajdhani College, University of Delhi, New Delhi -110015, India.

\*email - divyasingh1984@gmail.com

### Abstract

We study the effect of magnetic field and plasma density on the mechanism of terahertz generation for flat top laser pulse. Terahertz fields are computed after evaluation of the components of wakefield those are generated due to propagation of highly intense pulse in a uniform density magnetoactive plasma. The effect of flat top lasers on the magnitude of wakefield are investigated and further theoretical description is presented for emission of terahertz pulses. It is also observed that there is no wakefield produced in the direction of the external magnetic field whereas components of wakefield are observed in the direction parallel and perpendicular to the axis of propagation of laser perpendicular to the direction of magnetic field.

**Keywords:** Laser Plasma Interaction, Terahertz, Wakefield, Perturbation and Quasistatic approach.

### Introduction

Now a days Terahertz frequency domain of electromagnetic spectrum has been a very important and integrated part of technology based on their application as a potential field of research due to its viability. Terahertz radiation is nonionizing electro-magnetic radiation with submillimeter wavelength, which can be used non-invasively to several applications. These applications involve imaging, material characterization, topography, tomography, communication [1,2] etc. There exist many schemes of THz generation, which exploit traditional nonlinear optical methods of laser matter interaction. Laser-triggered solid-state based sources of THz radiation have been developed those are based on the switched photoconducting antennas [3] and on optical rectification of femtosecond lasers [4]. As for THz emission ultraintense strong lasers are required therefore material are not sustainable at such higher powers which leads to material breakdown, hence due to low damage limit of matter very small power THz pulses are obtained. These limitations and low conversion efficiency of mechanism are overcome by usage of plasma as a medium for interaction with lasers. THz emission from laser excited plasmas had been reported earlier [5] and was attributed to the excitation of radial plasma density oscillations at a plasma frequency resonant with the laser pulse duration. Alternatively, THz radiation can be generated from electron beams through various methods, including bending in a magnetic field (synchrotron radiation) [6] and by traversing a medium with a discontinuity in dielectric properties (transition radiation) [7, 8]. Our group also had obtained strong terahertz radiation with the use of supergaussian lasers in collisional plasma [9].

A laser wakefield is an electron plasma wave driven by the ponderomotive force of a laser pulse in plasma [10]. It has been studied intensively for the purpose of particle acceleration so far. Since the typical plasma oscillation frequency of these waves lies in the terahertz (THz) range, therefore the wakefield can potentially serve as a powerful THz emitter. On the other hand, even though the laser wakefield can be driven at high amplitudes and it seems that usually a plasma wave cannot be converted into an electromagnetic wave directly because of their different dispersion relations. But, it has been found from numerical simulations that intense radiation around the plasma frequency can be produced from the wakefield in inhomogeneous plasma [11]. Yoshii et al. [12] theoretically and Yugami et al. [13] experimentally have demonstrated the THz radiation generation when the Cerenkov wake is excited by a short laser pulse in a perpendicularly magnetized plasma. In an another study Sheng et al. [14] proposed a scheme in which a short laser pulse excites a large amplitude plasma wake field, which, in the presence of an axial density gradient, produces radiation at the plasma frequency ( $\omega_p$ ) via mode conversion.

Here in this paper, we revisit the mechanism for generation of terahertz radiation based on the wake field approach. We calculate amplitude of terahertz radiation from the wake fields produced by the propagation of highly intense ultrashort lasers through magnetised plasma of uniform density. Analytical calculations are carried out to evaluate the components of wakefield based on the approach of perturbative technique under quasistatic approximation. It is observed that there is no wakefield produced in the direction of the external magnetic field whereas components of wakefield are observed in the direction parallel and perpendicular to the axis of propagation of laser. It is found that the vertical component of wakefield excites the generation of electromagnetic radiation in the terahertz frequency range in the presence of transverse magnetic field.

### Schematic

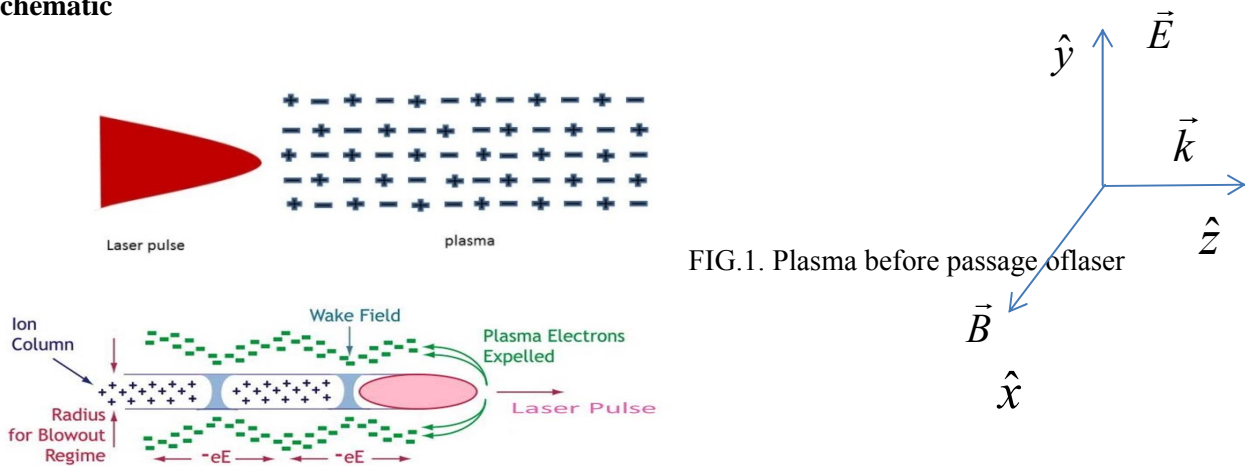


FIG.2 Wakefield generation in plasma on passage of laser [15].

### Calculation of Wake-Field and Terahertz Emission

We consider a propagation of flat top laser of frequency  $\omega$  and wave number  $k$ , in the  $z$  direction of gaseous plasma of uniform density  $n_0$ . The laser is polarized along the  $y$  direction.

The fields of the laser is assumed as

$$\vec{E} = E_0 \exp\left[-\left(\frac{y}{b_w}\right)^8\right] e^{i(kz-\omega t)} \hat{y}$$

Where,  $b_w$  is the beamwidth where the amplitude of the laser field becomes  $1/e$  times of its peak value.

$$\text{Equation of motion of electron fluid in plasma is } m \frac{\partial \vec{v}}{\partial t} = -e[\vec{E} + \frac{1}{c}\{\vec{v} \times \vec{B}\}] \quad (1)$$

$$\text{Equation of Continuity is } \frac{\partial n}{\partial t} + n_0(\vec{\nabla} \cdot \vec{v}) = 0. \quad (2)$$

We made quasistatic approximation and apply reductive perturbative approach for further analysis of the problem. The physical quantities are expanded to their higher orders using perturbative technique in orders of laser normalised amplitude parameter-  $a$  ( $= \frac{eE}{mc\omega} \ll 1$  approximated) such that

plasma density as  $n = n^{(0)} + an^{(1)} + a^2n^{(2)}$ , Plasma electron velocities as  $v = v^{(0)} + av^{(1)} + a^2v^{(2)}$  and plasma current densities as  $J = J^{(0)} + aJ^{(1)} + a^2J^{(2)}$ .

The expanded equation of motion and equation of continuity are solved to find out the zero and first order components of velocity and density as follows

$$\vec{v}_x^{(0)} = \vec{v}_z^{(0)} = 0, \vec{v}_y^{(0)} = \frac{eE_0}{mi\omega} e^{i(kz-\omega t)} \hat{y} \text{ and } n^{(0)} = n_0.$$

$$\vec{v}_x^{(1)} = 0, \vec{v}_y^{(1)} = \frac{eE_0\omega}{im[\omega^2 - \omega_c^2]} e^{i(kz-\omega t)} \hat{y},$$

$$\vec{v}_z^{(1)} = -\frac{eE_0\omega_c}{m[\omega^2 - \omega_c^2]} e^{i(kz-\omega t)} \hat{z}.$$

$$n_x^{(1)} = 0, n_y^{(1)} = \frac{kn_0\vec{v}_y^{(1)}}{(\omega - k\vec{v}_y^{(0)})}, n_z^{(1)} = \frac{kn_0\vec{v}_z^{(1)}}{\omega}.$$

While solving for first order, the zero order components of velocity and density are used and cyclotron frequency is represented as  $\omega_c = \frac{eB}{mc} \hat{x}$ , These nonlinear velocities are generated due to plasma nonlinear forces having  $x$ ,  $y$  and  $z$  components and produces density perturbations with their respective components. Due to the density perturbations in plasma, coupling of plasma oscillations with density fluctuations take place, that driven nonlinear currents.

These generated nonlinear currents are evaluated using zero and first order velocity and density component such as,  $\vec{J}_j^{(1)} = -(n_0 e \vec{v}_j^{(1)} + n_1 e \vec{v}_j^{(0)})$  where  $j = x, y, z$  component. These nonlinear currents are responsible for excitation of wakefields in magnetised plasma.

We have time dependent Maxwell's equations  $\vec{\nabla} \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$

$$(3) \vec{\nabla} \times \vec{B} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \varepsilon \frac{\partial \vec{E}}{\partial t} \quad (4) \text{Under Quasistatic Approximation (QSA), the laser pulse field}$$

variations are assumed to be time independent. We write all components of maxwell's equations in terms of transformed coordinates  $\xi = z - ct$  as laser propagates along z axes with phase velocity c

$$\frac{\partial \vec{E}_z}{\partial y} - \frac{\partial \vec{E}_y}{\partial \xi} = \frac{\partial \vec{B}_x}{\partial \xi}, \quad (3a) \quad \frac{\partial \vec{E}_x}{\partial \xi} - \frac{\partial \vec{E}_z}{\partial x} = \frac{\partial \vec{B}_y}{\partial \xi}, \quad (3b) \quad \frac{\partial \vec{E}_y}{\partial x} - \frac{\partial \vec{E}_x}{\partial y} = \frac{\partial \vec{B}_z}{\partial \xi}, \quad (3c)$$

$$\frac{\partial \vec{B}_z}{\partial y} - \frac{\partial \vec{B}_y}{\partial \xi} = \frac{4\pi}{c} \vec{J}_x - \frac{\partial \vec{E}_x}{\partial \xi}, \quad (4a) \quad \frac{\partial \vec{B}_x}{\partial \xi} - \frac{\partial \vec{B}_z}{\partial x} = \frac{4\pi}{c} \vec{J}_y - \frac{\partial \vec{E}_y}{\partial \xi}, \quad (4b)$$

$$\frac{\partial \vec{B}_y}{\partial x} - \frac{\partial \vec{B}_x}{\partial y} = \frac{4\pi}{c} \vec{J}_z - \frac{\partial \vec{E}_z}{\partial \xi} \quad (4c)$$

Eq (1) is further solved for first order under QSA in transformed coordinates as follows,

$$\frac{\partial \vec{v}_x^{(1)}}{\partial \xi} = \frac{e}{mc} \vec{E}_x \quad (5a) \quad \frac{\partial \vec{v}_y^{(1)}}{\partial \xi} = \frac{e}{mc} \vec{E}_y + \frac{\omega_c}{c} v_z^{(0)} \quad (5b) \quad \frac{\partial \vec{v}_z^{(1)}}{\partial \xi} = \frac{e}{mc} \vec{E}_z - \frac{\omega_c}{c} v_y^{(0)} \quad (5c)$$

using equations (3c) and (4c) and on substitution of  $J_z^{(1)}$  and (5c) we get

$$\left\{ \frac{\partial^2}{\partial \xi^2} + k_p^2 \right\} \vec{E}_z = -\frac{k_p^2 m}{e} \omega_c v_y^{(0)} \quad (6)$$

where  $k_p^2 = \frac{4\pi n_0 e^2}{mc^2}$  is plasma wave number . On solving equation (6) for magnetised plasma with boundary condition of laser propagation  $\{ \xi = 0, E_z = 0 \ \& \ \xi = L/2, E_z = 0 \}$ , where L is the length of the plasma, we obtain horizontal component of wake field

$$\vec{E}_z = -\frac{m\omega_c v_y^{(0)}}{e} \left[ 1 - \cos k_p \xi - \tan \frac{k_p L}{4} \sin k_p \xi \right] \quad (7)$$

We further obtain vertical components of wake fields in terms of the horizontal wake



$$\vec{E}_x = -\frac{1}{k_p^2} \frac{\partial^2 \vec{E}_z}{\partial \xi \partial x} \quad (8a)$$

$$\vec{E}_y = -\frac{1}{k_p^2} \frac{(\omega - kv_y^{(0)})}{\omega} \frac{\partial^2 \vec{E}_z}{\partial \xi \partial y} \quad (8b)$$

On further solving equation(8a & 8b) we obtain that there is no wakefield in the direction of external applied magnetic field i.e.

$$\vec{E}_x = 0, \quad (9a)$$

$$\vec{E}_y = \frac{8}{k_p} \frac{i\omega_c(\omega - kv_y^{(0)})E_0 e^{-(y/b_w)^8} e^{i(kz - \omega t)}}{b_w \omega} \left(\frac{y}{b_w}\right)^7 \times \left[ \sin k_p \xi - \frac{\tan k_p L}{4} \cos k_p \xi \right] \quad (9b)$$

It is evident from mathematical calculations that there is no wakes produced in the direction of external magnetic field but fields are presents in the direction of laser field as given by equation (9a & 9b). Horizontal component of wakefield gives rise to vertical component of wake in the direction of electric field of laser and this vertical component of field is responsible for generation of electromagnetic waves i.e. terahertz radiation in the direction perpendicular to the axis of propagation of laser. Generally the wakefield of lasers in plasma are used for the purpose of particle acceleration which is served by the horizontal component of wakefield and the vertical component is very small in magnitude for application purpose.

As we see clearly from equation (9b) that the y-component of the wakefield is responsible for generation of terahertz. We propose to make the use of laser, plasma properties along with the application of external magnetic field to enhance the magnitude of emitted wakefields and terahertz radiation. Y-component of wakefield is much larger for the flat top laser if optimised for beamwidth in the presence of external magnetic field.

## Result and Discussion

In our scheme, with employment of a laser pulse of flat top profile we tried to increase the y-component of the wake field than horizontal component by controlling beam width of laser pulse. This can be seen from equation (9b) that the amplitude of vertical wake field is directly depending on the

flatness and on the beam width of laser, therefore by controlling these laser parameters we can obtain higher magnitude wakefield and these y-component of wake field is responsible for generation of terahertz radiations hence higher amplitude terahertz waves may be emitted.

From fig-3 variation of emitted terahertz radiation from vertical component of wakefield is shown with beam width for flat top laser profiles, it is obvious from the figure that large magnitude of wakefield are excited for laser of low width therefore large amplitude of terahertz radiation is emitted.

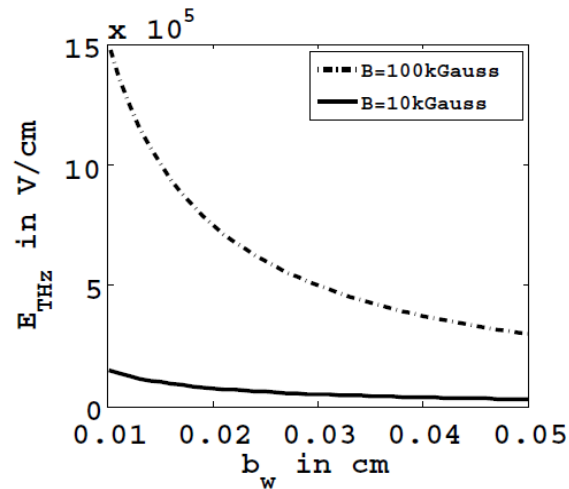


Fig.3. Variation of magnitude of emitted terahertz radiation for flat top laser with beam width  $b_w$  with variable external magnetic field, when  $\omega=2.4 \times 10^{14}$  rad/sec,  $\omega_p=2.0 \times 10^{13}$  rad/sec,  $y=0.45b_w$ .

The effect of external magnetic field is also analysed from fig-3. The increasing magnitude of applied magnetic field shows a direct and significant effect to the increment of the amplitude of the emitted terahertz radiation. This enhancement in terahertz field may be attributed of the cyclotron motion of plasma electrons that help to generate stronger plasma waves and hence strong nonlinear currents causing large wakefields.

The optimized value of transverse wakefield is also obtained from equation (9b) for the condition  $\xi = L/4$

$$\frac{\partial \bar{E}_y}{\partial \xi} = \frac{i8\omega_c(\omega - k \frac{eE_0}{m\omega})E_0 e^{-(y/b_w)^8} e^{i(kz-\omega t)}}{b_w \omega} \left( \frac{y}{b_w} \right)^7 \times \left[ \cos k_p \xi + \frac{\tan k_p L}{4} \sin k_p \xi \right] = 0 \quad (10)$$

Which is obtained from equation (10), that shows the vertical component of excited wakefield will be maximum.

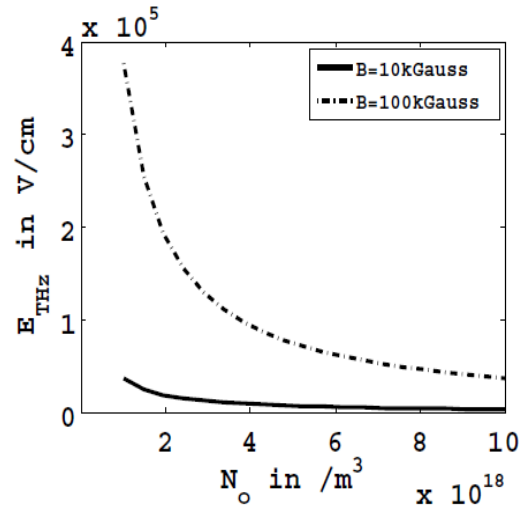


Fig.4. Variation of magnitude of emitted terahertz radiation with plasma density for variable magnetic field when  $b_w=0.05mm$ ,  $\omega=2.4 \times 10^{14} rad/sec$ ,  $\omega_p=2.0 \times 10^{13} rad/sec$  and  $y=0.45b_w$ .

The effect of plasma density on terahertz generation is discussed from fig-4. There is a critical density for maximum generated amplitudes of terahertz radiation. If density is increased above the critical value, leading to the decay of terahertz amplitude. Therefore it is concluded that this is the underdense plasma that supports the generation of strong terahertz radiation.

### Conclusions

The components of wakefield are computed using perturbation and quasistatic approach to their first order. It was observed that in the direction of external magnetic field there is no wake component observed whereas horizontal component of wakefield in the direction of laser propagation and vertical component of wakefield in the direction of laser field are realized. It was found that transverse component of wake is responsible for excitation of THz radiation, which can be enhanced by using lasers of lower beam width in the presence of external magnetic field.

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